MODAL TRUNCATION VS ISOBR

High-precision machines are often modelled as assemblies of flexible substructures, which preserves physical insight but leads to models that are too large for efficient dynamic analysis and control design. This article compares two model order reduction methods on a two-axis gantry: state-of-the-art substructure-based modal truncation (MT) with residualisation, and interconnected second-order balanced reduction (iSOBR), which accounts for the selected assembly input-output behaviour. Results show that iSOBR achieves similar input-output accuracy with 3.4 times fewer states and helps identify which parts of the machine are most relevant to the performance of interest.

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Introduction

Industrial precision machines are commonly built as interconnected systems: they are assemblies of structural components, actuators, interfaces and control elements. This modular structure is important not only for design and manufacturing, but also for modelling, since it preserves physical interpretation and allows these substructure models to be exchanged, reused or improved independently.

To predict the dynamic behaviour of interconnected systems, finite-element structural-dynamics models are often used. These models can become very large when geometrically complex, flexible substructures and interface dynamics are modelled in detail. Direct use of the full-order model in design iterations, controller synthesis or sensitivity studies is then impractical or even infeasible, so model order reduction is required [1].

In industrial practice, this is often tackled using component mode synthesis (CMS): each substructure model is reduced by selecting only eigenmodes with eigenfrequencies below a cut-off frequency and retaining static modes for the interface degrees of freedom. This approach facilitates system assembly and guarantees static accuracy [2].

Although CMS is robust and widely used, it does not always lead to a sufficiently reduced assembly model. Especially when many local substructure modes are present in the frequency range of interest, the reduced model may still be too large. Moreover, reducing substructures independently can remove dynamics that become relevant after integration. This motivates structure-preserving reduction methods, which reduce subsystem models while explicitly accounting for the selected assembly input-output dynamics [1] [3].

Model reduction of interconnected systems is an active research field. Recent works have developed structure-preserving balanced-reduction methods that not only retain modularity but also, depending on the method, second-order structure, stability and passivity [4]. Related interconnection-based model-reduction methods similarly treat the interconnection behaviour itself as a key property to preserve [5].

Gantry systems are a typical example where model-size challenges become apparent. They consist of many interconnected substructures, such as frames, stages, guides and supports, which can lead to complex assembly models with many local dynamics. Even after standard CMS reduction, the model state dimension often remains too high for efficient analysis. At the same time, the modular structure should be preserved to retain physical insight and enable substructure-level modifications.

The recently developed interconnected second-order balanced reduction (iSOBR) method of [4], which focuses on the accurate approximation of assembly input-output behaviour of interest, is a promising approach that addresses all these aspects.

To compare the iSOBR and a standard model-reduction method in the discussed context using an industrial use case, this article considers the Bosch Rexroth Gantry Demonstrator designed by MI-Partners [6], as shown in Figure 1.

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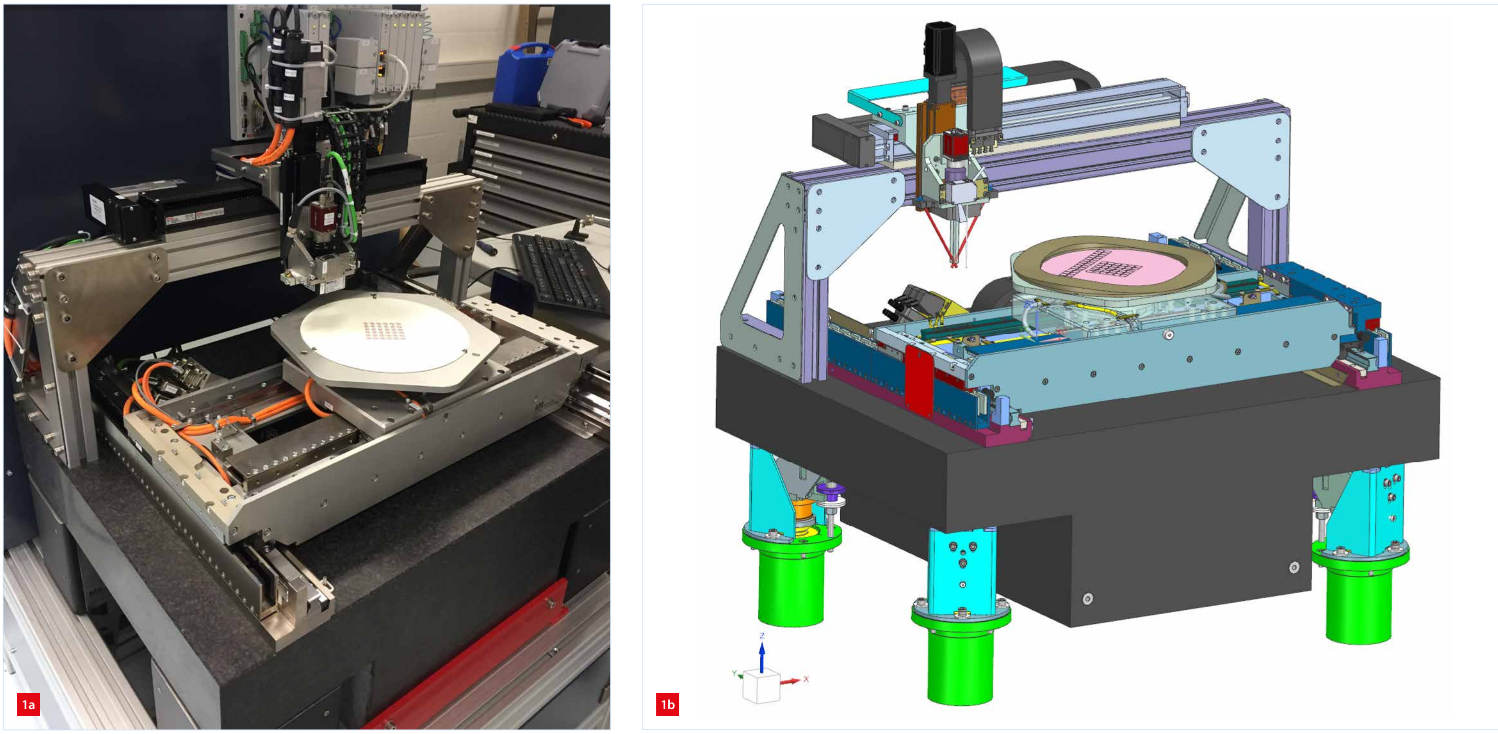
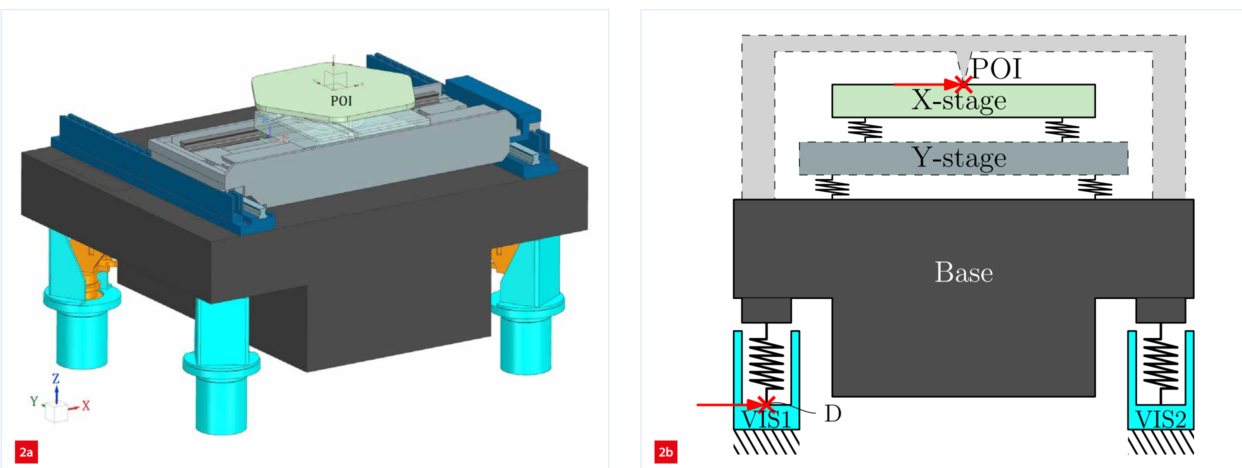


The gantry system considered in this article; the Bosch Rexroth Gantry Demonstrator designed by MI-Partners.
(a) Realisation.
(b) CAD model.

An XY-motion gantry use case

The gantry system shown in Figure 1 is a precision mechatronic system with X-, Y-translational and Z-rotational motion, integrated actuation, and a vibration-isolated support structure. The system combines lightweight structural components with a flexible suspension that decouples the machine from floor vibrations at low frequencies. As a result, the assembly model contains many relevant eigenmodes over a wide frequency range. This makes the demonstrator a representative use case: the model state dimension is large, while the interconnection structure should be retained for efficient and interpretable (re-)design and dynamic analysis [6].

For this article, the demonstrator is simplified to an XY-stage, removing the upper bridge with the Z-stage and locking the rotational Z-motion (see Figure 2). A point of interest (POI) is defined at the top of the table, where we monitor the relative X- and Y-displacements between the X-stage and a virtual point on a virtual rigid frame (indicated in light grey in Figure 2) connected to the base. Two external input sets are considered: (i) an external process X- and Y-force applied at the POI, in the original system representing an internal force between the bridge and the X-stage, and (ii) an external disturbance X- and Y-force applied at the bottom of the compression spring of the first vibrational isolator (VIS1), representing floor vibrations.



Overview of the demonstrator.
(a) Simplified CAD model.
(b) 2D schematic drawing; the X- and Y-force inputs (indicated in red) act on the POI at the top of the X-stage and at the point D, on VIS1.

As shown in Figure 2, the system model consists of seven substructures: the four VIS models, which consist of a stiff leg and a large, flexible compression spring; the granite base including the linear guides and Y-stage motors; the Y-stage; and the X-stage, including the (fixed) rotary table. Each substructure is modelled separately, after which they are connected to form the system (assembly) model. This approach is discussed in more detail in the following section.

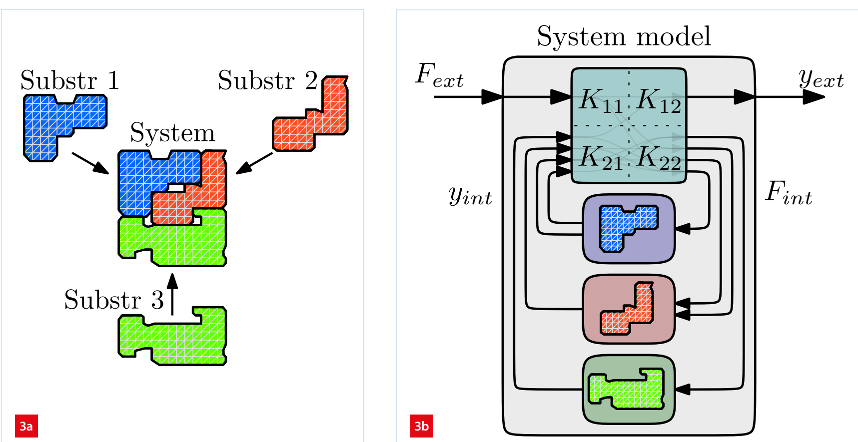
Modelling using substructures

In a modular design process, each substructure is not only designed, but also modelled, separately. To predict the performance and properties of the full, interconnected system, a dynamic model is constructed by interconnecting all subsystem models. The introduced interconnection structure (using a so-called connection matrix K) enables modular usage, such as the independent evaluation of sub-structure models, straightforward replacement of substructure models and in-depth understanding of the contribution of substructures to the interconnected system's behaviour.

In this article, each substructure is initially modelled and analysed using the finite-element package Ansys. For each substructure, undamped eigenfrequencies and eigenmodes are calculated up to a cut-off frequency of 10 kHz, which can then be used to create second-order modal models. The number of kept eigenmodes per substructure is given in Table 1.

Table 1
Number of eigenmodes and eigenfrequencies calculated per substructure, including the maximum calculated eigenfrequency.

Substructure	Number of modes	Max. eigenfrequency (Hz)
VIS (1-4)	4 x 96	9,550
Base	185	9,984
Y-stage	142	9,959
X-stage	84	9,965
Total	**795**	-



System model consisting of three substructures.
(a) Schematic drawing.
(b) Schematic representation of the internal and external interconnections, defined by the connection matrix K.

To obtain the system (assembly) model, the connection matrix K relates external force inputs F_{ext} on the assembly and internal displacement outputs y_{int} to (selected) external displacement outputs y_{ext} and internal force inputs F_{int}, as visualised in Figure 3. Matrix partition K_{22} contains internal interface stiffnesses between substructures, such as those representing the ball bearings of the linear guides. These ball bearings are modelled using linear stiffnesses of 10^8 N/m, whereas each VIS, which is internally flexible, is connected to the base with very stiff translational and torsional springs of 10^9 N/m and 10^7 N/m, respectively. In addition, we assume the system's X- and Y-stage are in control at some operational point using a basic proportional controller, such that there is also a 10^6 N/m stiffness at the linear motors in the motion direction.

By connecting the substructures using all these elastic connections, a system model with 795 modes (1,590 states) is obtained. Although this model is already severely reduced compared to the original finite-element model (FEM) of 641,355 degrees of freedom, 795 modes is still considered too high; below, we will refer to the 795-modes model as the unreduced model.

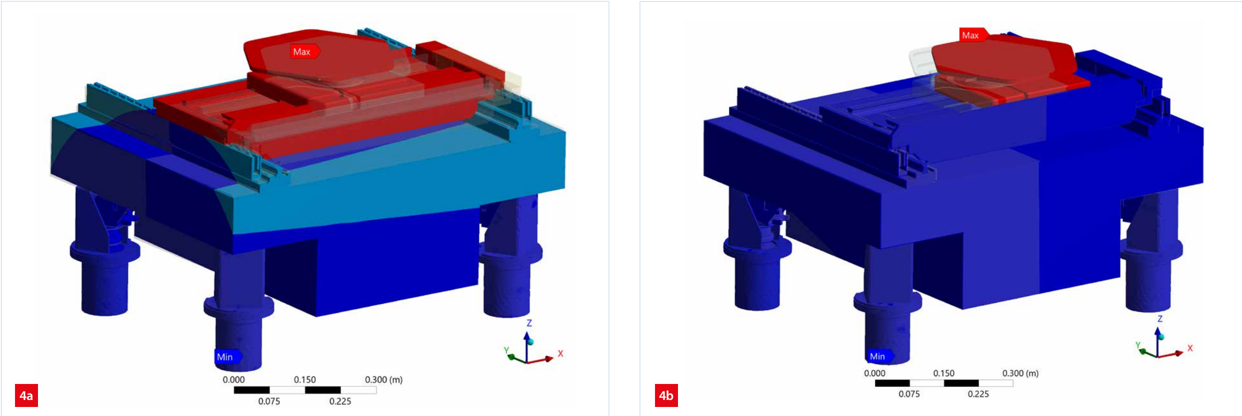
To evaluate the reduction methods, two 2x2 transfer matrices are considered. The first relates process forces applied at the POI to the relative X- and Y-displacement at the POI. The second relates disturbance forces applied at VIS1, representing floor vibrations, to the same relative displacements at the POI. These transfers represent the performance measures considered throughout the remainder of this article.

Before discussing model reduction, it is useful to inspect the dominant dynamics of the unreduced assembly model. Figure 4 shows two important assembly modes, corresponding to motion of the Y-stage and the X-stage along their respective linear guides.

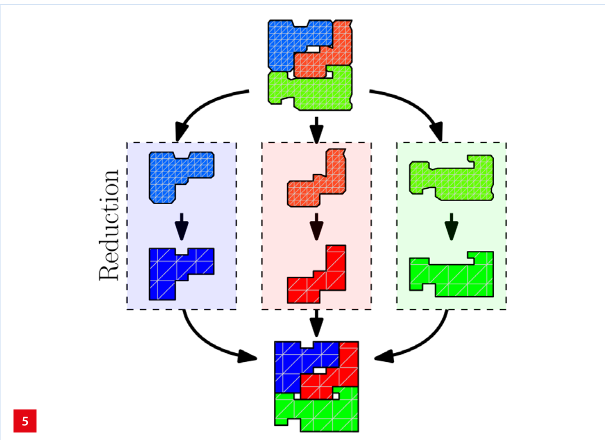
Structure-preserving model order reduction

As already mentioned, the structure of the system model is paramount for its modular modelling ability. Straightforward reduction of the system model, without considering its internal structure, destroys this modularity. To this end, reduction is also often applied, in a modular fashion, to the substructures separately, as shown in Figure 5.

In the field of structural dynamics, one typically uses component mode synthesis (CMS) approaches such as Craig-Bampton or Hintz-Herting [7] [8]. These methods retain eigenmodes up to a selected cut-off frequency and preserve low-frequency stiffness effects through static modes [2].



Mode shapes of the complete FEM assembly model.
(a) Dominant Y-stage motion along the linear guides (49 Hz).
(b) X-stage motion along the linear guides (59 Hz).



Schematic representation of structure-preserving reduction, where each substructure is reduced separately and the reduced interconnected model is built from the reduced substructures.

In this work, we adopt a related baseline approach, referred to as modal truncation with residualisation. High-frequency free-interface substructure eigenmodes are removed, while their contribution to the low-frequency (static) compliance is retained in the direct-feedthrough (D) matrix through residualisation [9].

Whereas CMS-like approaches describe global dynamics in terms of substructure eigenmodes, interconnected second-order balanced reduction (iSOBR) [4] ranks balancing modes by their relevance to the selected assembly input-output behaviour. It reduces each substructure separately and thereby preserves the modular interconnection structure of the system model. To this end, it uses the assembly controllability and observability Gramians, which quantify how strongly states are influenced by external inputs and observed in external outputs [9]. The model is subsequently transformed into a balanced representation, where the importance of each component balancing mode is quantified by a structured Hankel singular value (sHSV) [10], which provides a measure of the mode energy. The component balancing modes with the largest sHSV are retained, while the remaining modes are removed using residualisation, as in the baseline method.

Reducing the gantry model

Reduction set-up

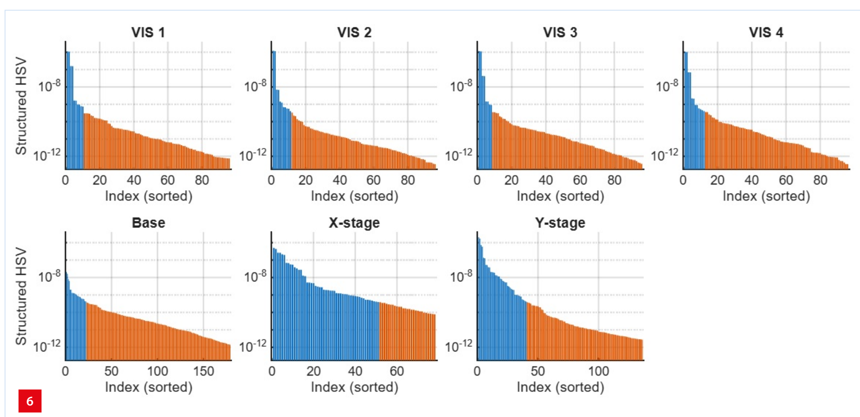
The model is reduced in a structure-preserving manner, where each substructure is reduced individually, using both modal truncation and iSOBR, in combination with residualisation (see previous section).

The following reductions are considered:

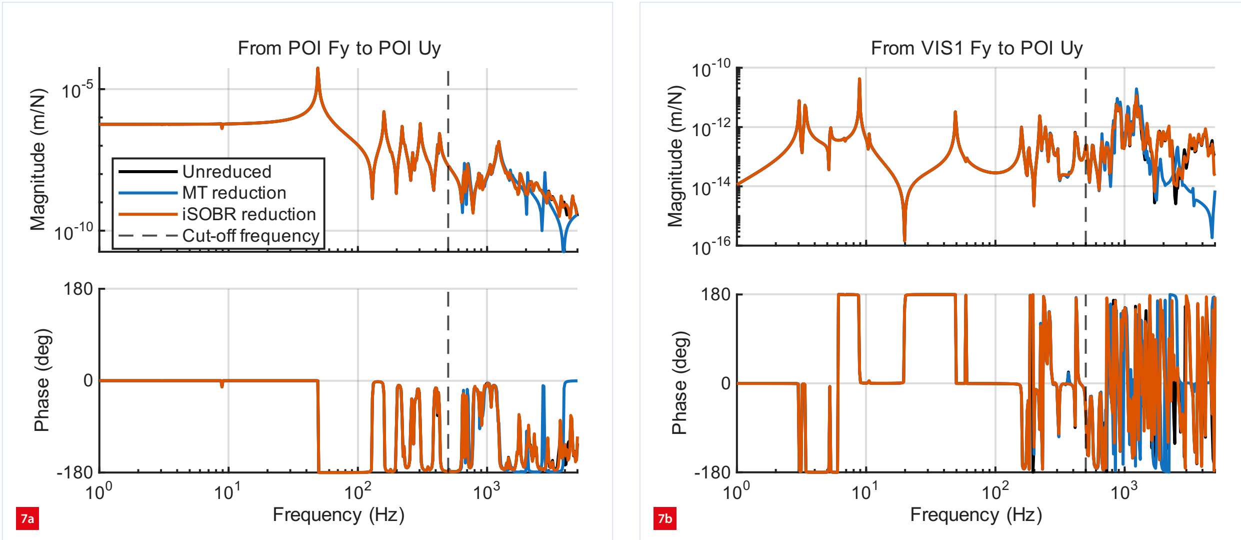
1. Modal truncation with a 1,500 Hz cut-off frequency, resulting in a 344-state model.
2. iSOBR using the POI-to-POI transfer (process force to relative POI displacement), also resulting in 344 states.
3. iSOBR using the VIS1-to-POI transfer (VIS disturbance force to relative POI displacement), also resulting in 344 states.
4. iSOBR using the POI-to-POI transfer, reduced further to 100 states.

POI-to-POI results

To evaluate performance, we first consider the POI-to-POI transfer, relating process forces in the horizontal X- and Y-direction to the corresponding relative motion of the POI. This transfer is also used for the Gramian computation in iSOBR. The model orders of the resulting reduced models are given in Table 2, which indicates that the reduced system order has become 344 for both modal truncation (3rd column) and



Structured Hankel singular values per substructure for the POI-to-POI transfer. These sHSVs are used by iSOBR to select the retained balancing modes per substructure, using a cut-off of $3.9{\cdot}10^{-10}$; blue indicates retained modes and red indicates residualised modes.



FRFs of the transfer from force to relative POI displacement in the Y-direction, according to the unreduced model and the models reduced to 334 states by modal truncation (MT) and iSOBR. Note that the iSOBR-reduced transfer (red) almost completely overlaps the unreduced transfer (black).
(a) Process force.
(b) Disturbance force

Table 2
Reduced order (number of states) per substructure for modal truncation and the three considered iSOBR reductions.

Substructure	Unreduced order	Reduced order			
		Modal	iSOBR (POI)	iSOBR (VIS -> POI)	iSOBR (POI)
VIS1	192	60	20	104	8
VIS2	192	60	22	18	4
VIS3	192	60	16	20	8
VIS4	192	60	24	24	8
Base	370	32	56	68	12
Y-stage	284	38	92	66	28
X-stage	168	34	114	44	32
Total system	1,590	344	344	344	100

iSOBR (4th column). The reduced order per substructure, however, is very different: compared to modal truncation, the VIS models are reduced much more aggressively. Inspecting the sHSV in Figure 6, used to determine the reduced orders with iSOBR, the values related to the X- and Y-stage are relatively large, whereas those related to the VIS models are relatively small. This indicates that the stage dynamics are most important for the selected external input-output behaviour, while the VIS models, because they contain a lot of local dynamics, contribute less.

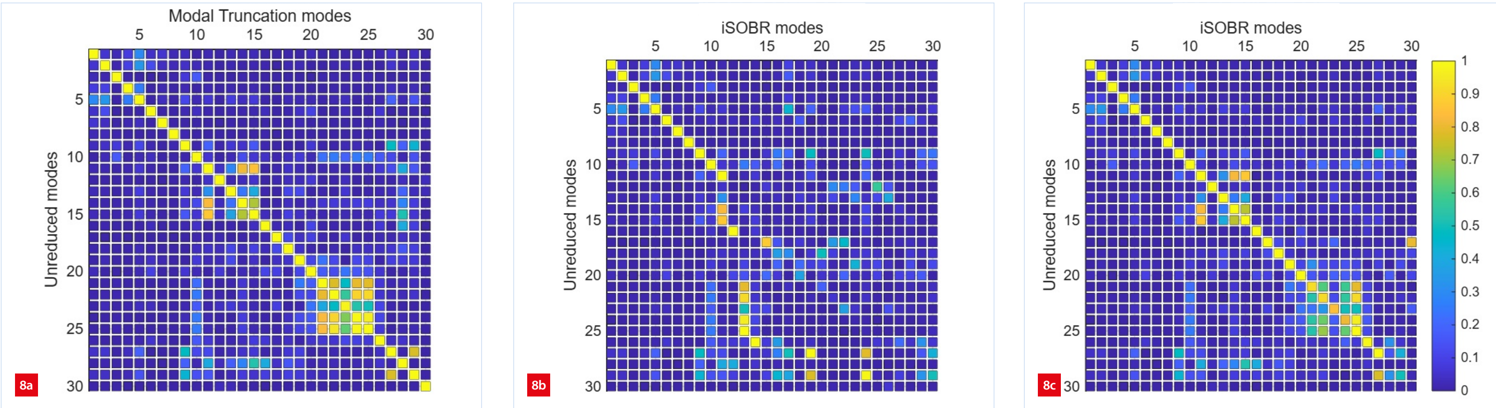
The resulting distribution of states by iSOBR, allocating most states to the stage models, agrees with physical intuition. The improved distribution, focusing on the dynamics that are most relevant for the external input-output behaviour, enables iSOBR to achieve a very accurate FRF (frequency response function) approximation, as shown in Figure 7a.

Intuitively, the VIS dynamics seem of limited importance for the process transfer and could be approximated by a static spring in this relative transfer; iSOBR does not exploit this, as it does not account for the residualisation approach in which the stiffness contribution of eliminated modes is preserved in the D-matrix. As a result, the first few sHSVs of the VIS models in Figure 6 remain large, and iSOBR retains several low-frequency modes mainly capturing spring-stiffness effects.

VIS1-to-POI results

To demonstrate how the Gramians and their substructure sHSVs change (and therefore indicate different substructure reductions) for other external input-output combinations, we also consider the disturbance input at the bottom of the spring of VIS1 (see also Figure 2) to the relative displacements at the POI. In this case, the distribution of states differs significantly from the previous case, as shown in the 5th column of Table 2, where VIS 1 now gets the largest number of states to capture the spring dynamics correctly. Figure 7b again shows that iSOBR closely matches the unreduced FRF and outperforms modal truncation. The close match suggests that the sHSV provides a good indication for allocating substructure reduction levels depending on the specific external input-output behaviour of interest.

Modal truncation and iSOBR can also be compared with the unreduced model using the modal assurance criterion (MAC), which quantifies similarity of structural eigenmodes by a value between 0 and 1. The MAC matrices, given in Figure 8, indicate that the model reduced by the iSOBR method (based on the POI input to POI output) does not contain several higher eigenmodes that both the modal truncation method and iSOBR (based on VIS input to POI output) retain. This indicates that these modes might be less relevant for the POI-to-POI transfer. Modes 16-18, for example, are all modes of the



MAC matrices comparing the structural eigenmodes of the three reduced models with those of the unreduced model. A value close to one indicates high similarity between two modes, whereas a value close to zero indicates low similarity.
(a) Modal truncation modes.
(b) iSOBR modes (POI-to-POI transfer).
(c) iSOBR modes (VIS-to-POI transfer).

VIS spring, which are hardly relevant for the POI-to-POI transfer, but are relevant for the VIS-to-POI transfer.

Accuracy versus model size

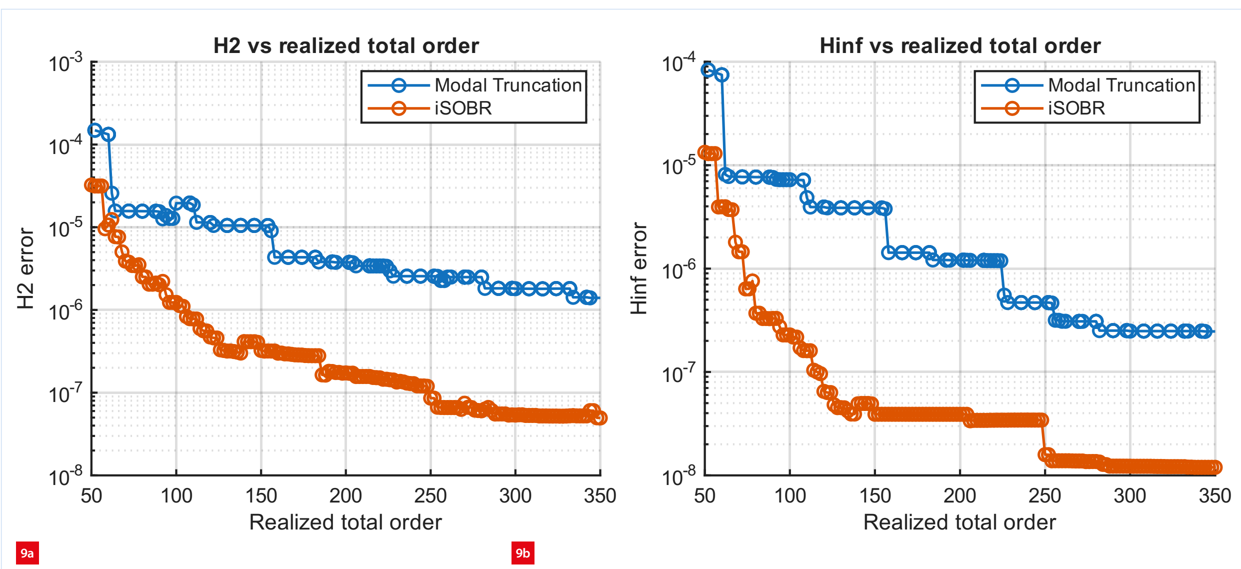
While visual FRF comparison (see Figure 7) typically gives direct qualitative information on input-output accuracy, it does not judge the accuracy quantitively. To this end, we also evaluate the approximate H_2- and H_∞-norms of the 2x2 POI-to-POI FRF error matrix E. These are based on the frequency response data (evaluated at radial frequencies ω_i) as follows:

$$||E||_2 = \left(\frac{1}{2\pi} \sum_{i=1}^{n_\omega - 1} \mathrm{trace}\big(E^*(j\omega_i)E(j\omega_i)\big)\ (\omega_{i+1} - \omega_i) \right)^{1/2}$$

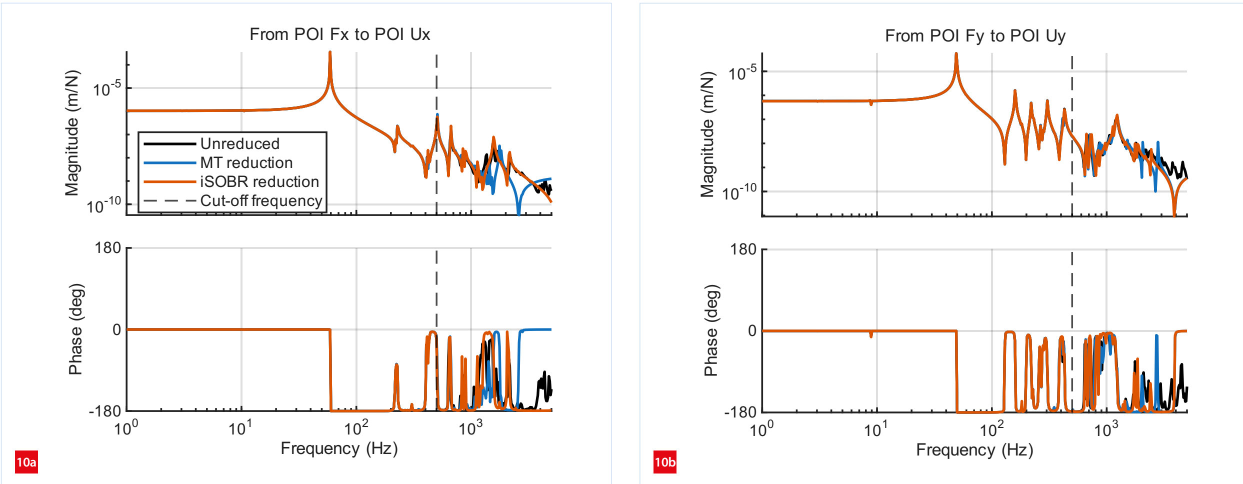
$$||E||_\infty = \max_i(\bar{\sigma}(E(j\omega_i))$$

Here, $\bar{\sigma}(.)$ indicates the largest singular value of $E(j\omega_i)$. Figure 9 shows the error norms for various orders of the reduced system model, which indicates that iSOBR can achieve a similar accuracy with approximately 100 states as modal truncation (MT) achieves with 344 states.

To validate this, the iSOBR reduction is repeated with a total reduction order of 100 states, which results in a distribution of states over the substructure models as shown in the last column of Table 2. The FRF of the resulting 100-states iSOBR model is compared to the unreduced model and the 334-states MT-reduced model in Figure 10, which indeed shows a similar accuracy.



Error for various orders of the reduced system model for reduction by both modal truncation and iSOBR.
(a) H_2-error.
(b) H_∞-error.



Collocated POI FRFs according to the unreduced model (black) and the models reduced by modal truncation (MT) to 334 states (blue) and by iSOBR to 100 states (red).
(a) X-direction.
(b) Y-direction.

Conclusion

The results show that iSOBR is not only effective in reducing the gantry model, but also provides useful insight into the importance of individual substructure dynamics for selected system-level input-output behaviour. iSOBR indicates, by means of the structured Hankel singular values, which parts of the model are most relevant for the FRFs of interest. Indeed, in general, different choices for system-level inputs and outputs will lead to a different distribution of retained balanced states over the substructures.

Compared to modal truncation, iSOBR enables a much higher level of model reduction. In the considered case study, a model with 100 states achieves similar accuracy as the 344-states model obtained by modal truncation. This confirms that CMS and related modal approaches are conservative: they retain all eigenmodes up to a chosen cut-off frequency, regardless of whether these modes significantly contribute to the selected global input-output behaviour, or not.

A promising direction for future research is to investigate if iSOBR can be used not only as a reduction method, but also as a modelling tool. The structured Hankel singular values can help identify which substructures or modes are truly relevant for a given system-level performance measure, and may therefore guide modelling choices such as where a more detailed modelling (and maybe even design) effort is required.

REFERENCES

[1] N. van de Wouw, *et al.*, "Structure-preserving model reduction of interconnected dynamical systems", *Annu. Rev. Control*, vol. 60, 2025, *doi: 10.1016/J.ARCONTROL.2025.101016*.

[2] R. Craig, Jr., "Coupling of substructures for dynamic analyses - An overview", in *41st Structures, Structural Dynamics, and Materials Conference and Exhibit*, Reston, VA, USA: American Institute of Aeronautics and Astronautics, Apr. 2000. *doi: 10.2514/6.2000-1573*.

[3] L. Poort, "Model Order Reduction of Interconnected Dynamical Systems: Efficient Preservation of Accuracy, Modularity and Stability", Eindhoven University of Technology, Eindhoven, 2025.

[4] L. Poort, *et al.*, "Structure-Preserving Approximate Balanced Reduction of Interconnected Structural-Dynamics Models", *2025 IEEE Conference on Control Technology and Applications, CCTA 2025*, pp. 660-666, 2025, *doi: 10.1109/CCTA53793.2025.11151314*.

[5] L.A.L. Janssen, *et al.*, "Modular model reduction of interconnected systems: A robust performance analysis perspective", *Automatica*, vol. 160, 2024, *doi: 10.1016/j.automatica.2023.111423*.

[6] MI-Partners, "Gantry stage fair demonstrator", accessed Jun. 16, 2026, *www.mi-partners.nl/gantry-stage*.

[7] R. Craig and M.C.C. Bampton, "Coupling of substructures for dynamic analyses", *AIAA Journal*, vol. 6 (7), pp. 1313-1319, 1968, *doi: 10.2514/3.4741*.

[8] D.N. Herting, "A general purpose, multi-stage, component modal synthesis method", *Finite Elements in Analysis and Design*, vol. 1 (2), pp. 153-164, 1985, *doi: 10.1016/0168-874X(85)90025-3*.

[9] A.C. Antoulas, *Approximation of Large-Scale Dynamical Systems*, SIAM, Philadelphia, PA, USA, 2005. *doi: 10.1137/1.9780898718713*.

[10] H. Sandberg and R.M. Murray, "Model reduction of interconnected linear systems", *Optim. Control Appl. Methods*, vol. 30 (3), pp. 225-245, 2009, *doi: 10.1002/oca.854*.