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ACTIVE AND PASSIVE DAMPING BASED ON PIEZOELECTRIC ELEMENTS – CONTROLLABILITY ISSUES –

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Abstract

Piezoelectric elements are widely used for damping micro-vibrations in mechanical structures. Active damping can be realised robustly by means of collocated actuator-sensor-pairs, controlled so as to extract vibration energy. Excellent damping performance is possible as long as sufficient controllability is assured. Passive damping based on a single piezoelectric element can be realised by the use of a resistive shunt. Damping performance however is very poor, due to the lack of controllability. A single piezoelectric element can also be used actively, in a self-sensing configuration, so as to improve controllability and damping performance. The non-linear capacitance of a piezoelectric element however endangers the robustness of the control scheme.

1 Introduction

The design of machine frames is often aimed at maximising the stiffness and minimising the mass. The issue of *damping* within machine frames is usually simply neglected during the design phase. An important reason for this is the fact that, unlike stiffness and mass properties, damping is hardly manageable. Especially within *high-precision* machines, typical unpredictable damping phenomena like friction, hysteresis and micro-slip have to be ruled out. As a consequence, high-precision machine frames are typically very lightly damped, which in turn gives rise to persistent vibrations (Van Schothorst 1999; Koster 2000).

In case vibrations in a machine frame turn out to be of the same order of magnitude as the desired accuracy, damping is often realised in an ad-hoc fashion, e.g., by means of a tuned mass damper or via layers of highly dissipative material. An important problem with this kind of damping solutions is the increased weight of the total structure. It is for this reason that, especially in the field of space and aircraft applications, damping methods based on *piezoelectric elements* have gained a lot of research interest in the past decades (Joshi 1989; Preumont 1997).

In the context of vibration control of high-precision machines, research at the Drebbel Institute of the University of Twente is aimed at the use of piezoelectric elements within a so-called ‘Smart Disc’. A Smart Disc is envisioned as an active structural element, consisting of a piezoelectric position actuator collocated with a piezoelectric force sensor and control and amplifier electronics. By inserting Smart Discs at appropriate locations in a high-precision machine frame and application of an appropriate control law, the effective damping of the frame may be improved significantly (Holterman and De Vries 2000).

With respect to the inherent actuator-sensor-collocation within a Smart Disc, it may be worth considering the use of a *single* piezoelectric element both as actuator and sensor, which may be regarded as ‘ultimate’ collocation. Furthermore, when damping is simply regarded as dissipation of vibration energy, it may be worth considering a *passive* implementation of the control law, by shunting such a single piezoelectric element with a passive electric circuit. Therefore, in this paper we will compare the damping performance of three closely related concepts:

(i) active damping with a collocated piezoelectric actuator-sensor-pair,
(ii) active damping with a single piezoelectric element (‘self-sensing actuator’),
(iii) passive damping with a single piezoelectric element.

The organisation of the paper is as follows. Section 2 describes the piezoelectric element to be used as sensor or actuator within a Smart Disc. Section 3 and 4 subsequently deal with the robustness and the performance of passivity-based vibration control schemes for collocated actuator-sensor-pairs. In section 4, which discusses controllability issues, crosstalk between the actuator and the sensor will appear to play a crucial role. In section 5, crosstalk will also appear to be a dominant phenomenon within a single piezoelectric element, limiting the achievable damping performance, be it by active or passive control. For that reason, section 6 briefly deals with crosstalk compensation, in order to improve damping capabilities. The overall conclusions that can be drawn from this analysis on the damping performance of the concepts (i)-(iii) will be summarised in section 7.

2 Piezoelectricity

In the introduction it was already stated that both the actuator and the sensor in a Smart Disc are to be made of *piezoelectric* material. Piezoelectricity is ability to convert mechanical energy into electrical energy and vice versa. The *direct* piezoelectric effect is that such a material, when subjected to mechanical stress, generates an electrical charge proportional to that stress. The *inverse* piezoelectric effect is that the same material becomes strained when an electric field is applied, the strain again being proportional to the applied field. Piezoelectric sensors obviously exploit the direct effect, whereas piezoelectric actuators rely on the inverse effect.

In order to correctly describe the deformation of a material, in-plane as well as shear stresses and strains in all three dimensions need to be considered, and use should be made of tensor quantities. For piezoelectric elements to be used in vibration control applications however, interest is usually in the behaviour of the material in a single direction. Restriction of the analysis of piezoelectricity to the direction of interest considerably simplifies the equations involved, as the material properties can then simply be expressed as scalars rather than as tensors (Jaffe *et al*. 1971; Waanders 1991; Preumont 1997).

When the analysis is restricted to a single direction, the behaviour of a piezoelectric element is often described by a linear relation:

$$\begin{bmatrix} x \\ q \end{bmatrix} = \begin{bmatrix} \left(k_m^U\right)^{-1} & d \\ d & C_{el}^F \end{bmatrix} \begin{bmatrix} F \\ U \end{bmatrix} \tag{1}$$

with

- force applied to the piezoelectric element F [N]
- expansion of the piezoelectric element x [m]
- voltage applied to the piezoelectric element U [V]
- charge built up in the piezoelectric element q [C]
- stiffness of the piezoelectric element at constant voltage k_m^U [N/m]
- capacitance of the piezoelectric element at constant force C_{el}^F [C/V]
- piezoelectric charge constant d [C/N = m/V]

In order to describe a charge-controlled piezoelectric position actuator or a voltage-generating piezoelectric force sensor, eq. (1) may be rewritten to:

$$\begin{bmatrix} x \\ U \end{bmatrix} = \begin{bmatrix} \left(k_m^q\right)^{-1} & g \\ g & \left(C_{el}^F\right)^{-1} \end{bmatrix} \begin{bmatrix} F \\ q \end{bmatrix} \tag{2}$$

with

- stiffness of the piezoelectric element at constant charge $k_m^q = \dfrac{k_m^U}{1-k^2}$ [N/m] (3)
- piezoelectric voltage constant $g = \dfrac{d}{C_{el}^F}$ [V/N = m/C] (4)
- coupling factor $k = d\sqrt{\dfrac{k_m^U}{C_{el}^F}}$ [–] (5)

In case of a *force sensor*, one may measure the voltage while keeping the charge at zero. The force can then be derived directly from the measured voltage (figure 1a):

$$U_{sens} = g_{sens} F_{sens} \Rightarrow F_{sens} = \frac{U_{sens}}{g_{sens}} \tag{6}$$

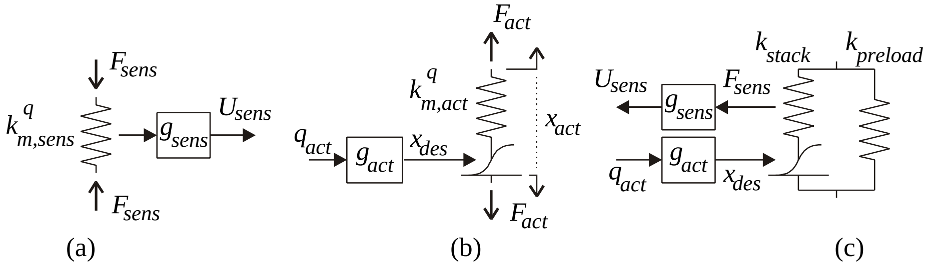

Figure 1 (a) Voltage-generating piezoelectric force sensor
(b) Charge-controlled piezoelectric position actuator
(c) Smart Disc: preloaded piezoelectric actuator-sensor-stack

In case of a *position actuator*, one may consider the displacement to be built up of a 'desired' displacement x_{des} (as if the actuator were unloaded, i.e., $F_{act} = 0$) and a contribution that is related to the force as experienced by the actuator, due to the fact that it is employed in a certain mechanical structure (figure 1b):

$$x_{act} = x_{des} + \frac{F_{act}}{k^q_{m,act}} \quad \text{with} \quad x_{des} = g_{act} q_{act} \tag{7}$$

Combining a piezoelectric force sensor and a piezoelectric position actuator yields the basis for a Smart Disc, as shown in figure 1c. In Smart Disc experiments performed so far, the actuator and sensor were stacked upon each other. The resulting piezoelectric stack can be modelled by a single stiffness:

$$k_{stack} = \frac{k^q_{m,sens} k^q_{m,act}}{k^q_{m,sens} + k^q_{m,act}} \tag{8}$$

As piezoelectric material in general should not be exposed to tensile forces, a Smart Disc may need to be equipped with a preload element, incorporated in figure 1c as a stiffness: $k_{preload}$. The influence of the preload element on the damping *performance* of a collocated actuator-sensor-pair will be discussed in section 4. The next section however first deals with the damping *robustness* of a collocated actuator-sensor-pair.

3 Collocation

From literature it is known that active damping can be robustly achieved by closing a single-input single-output (SISO) control loop between a collocated actuator-sensor-pair, as the controller then inherently imposes a relation between two variables that may be considered power-conjugated, at least within the controller bandwidth. This implies that the control law may be easily devised so as to guarantee the dissipation of energy from the vibrating structure, i.e., effectively provide damping to it. The robustness of such an active damping scheme, which is a simple form of *passivity-based control*, stems from the fact that it does not require detailed knowledge of the system (Preumont 1997).

With respect to the position actuator and the collocated force sensor in a Smart Disc, a passivity-based controller would be based on the use of an integrator in the feedback loop (*IFF*: Integral Force Feedback; Preumont 1997), as visualised in figure 2. The integrator may be considered to produce the actuated position x_{des} from a velocity v_{des}, which may in turn be regarded as a new variable to be controlled. As long as the

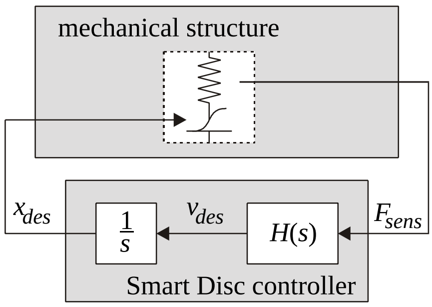

Figure 2 Illustration of energy dissipation based on Integral Force Feedback

dynamics between the actuator and the sensor may be neglected, the actuated velocity and the measured force may be considered as power-conjugated variables: their product denotes the power extracted from the mechanical structure:

$$P_{mech,out} = F_{sens} v_{des} \tag{9}$$

This implies that $H(s)$ in figure 2 may simply be chosen as a gain:

$$v_{des} = K \cdot F_{sens} \tag{10}$$

as in that case the power extracted from the mechanical structure is given by:

$$P_{mech,out} = K \cdot F_{sens}^2 \tag{11}$$

which for $K > 0$ never takes a negative value. As a result, energy is continuously being extracted from the mechanical structure, i.e., the vibrations are robustly damped.

In order to take, in addition to this energy-based point of view, a quick look from a control theory perspective, let's consider a simple two-mass system, in which a Smart Disc can be incorporated at two locations (figure 3). It is assumed that the uncontrolled two-mass systems show identical vibration modes: a lower vibration mode, corresponding to in-phase movement of the masses, and a higher vibration mode, corresponding to out-of-phase movement of the masses. The nice property of collocated actuator-sensor-pairs that enables robust active damping, can conveniently be seen from the pole-zero-maps of the open loop frequency responses (figure 4):

$$H_{mech}(w) = \frac{F_{sens}(w)}{x_{des}(w)} \tag{12}$$

exhibiting the alternating pole-zero-patterns observed along the imaginary axes. These alternating pole-zero-patterns are typical for collocated actuator-sensor-pairs and they can be shown to be excellent for robust active damping. To that end the lower plots in figure 4 incorporate an IFF-controller, which adds a pole close to the origin of the pole-zero-map (not a pure integrator, in order to prevent saturation due to e.g. drift):

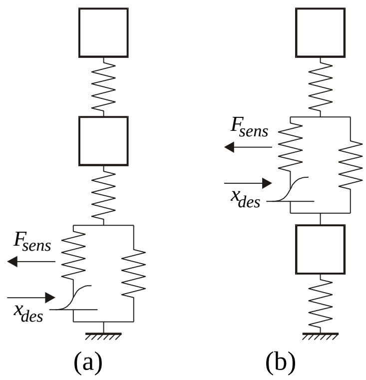

Figure 3 (a) Two-mass system supported by a Smart Disc
(b) Two-mass system with a Smart Disc in-between

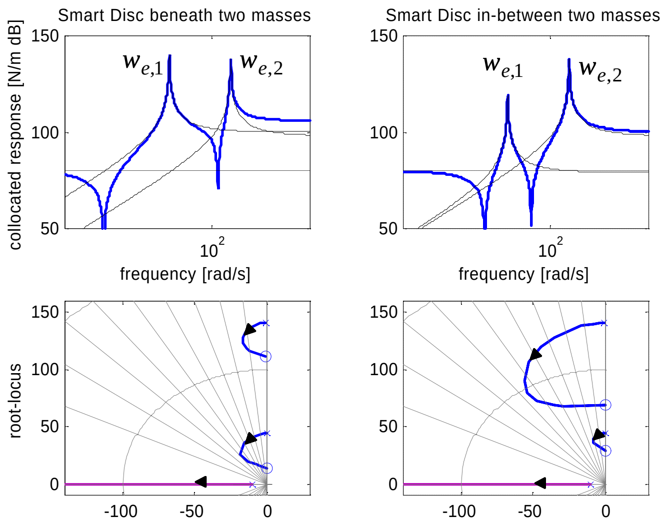

Figure 4 Frequency responses and IFF-root-loci for the structures of figure 3

$$H_{IFF}(s) = \frac{K}{s+a} \tag{13}$$

From the root-loci of the IFF-controlled structures, it can be seen that, due to the alternating pole-zero-pattern, in combination with the single controller pole close to the origin, for increasing IFF-gain K all poles are drawn into the left half of the *s*-plane, which implies that all resonances are damped.

Note that throughout this passivity-based analysis, apart from ignoring dynamics between actuator and sensor, we have assumed perfect actuator- and sensor-electronics, at least within the controller bandwidth. Note furthermore that, instead of IFF, more dedicated control is possible, enabling higher performance in theory, but at the cost of a loss of robustness in practice. This issue will briefly be addressed in section 5 and 6. The next section however first deals with a closely related issue: controllability.

4 Controllability

An important notion in any control problem is *controllability*, by which we mean the extent to which an actuator-configuration is able to change the behaviour of the system to be controlled. An equally important notion is *observability*, i.e., the extent to which a sensor-configuration is able to monitor this behaviour. Obviously, in the case of collocated actuator-sensor-pairs, controllability is dual to observability (Preumont 1997).

In this section we will evaluate the controllability associated with the collocated frequency response (eq. (12), upper plots in figure 4). In this respect it is important to be aware of the various contributions which together constitute this response. First of all one has to distinguish the *static* contribution, referred to as ‘crosstalk’, from the *dynamic* contributions. In the upper plots in figure 4 the straight line indicates the static crosstalk. For the Smart Discs in figure 3, the static crosstalk is caused by the preload stiffness, according to:

$$k_0 = \frac{k_{preload} k_{stack}}{k_{preload} + k_{stack}} \tag{14}$$

The preload contribution clearly manifests itself in the low-frequency range, resulting in a non-zero lowest anti-resonance frequency.

The dynamic contributions in the collocated response stem from the various vibration modes in the structure. It can be shown that each mode *j* adds a term of the form:

$$H_{dyn,j}(s) = l_j^2 k_j \frac{s^2}{s^2 + w_{e,j}^2} \tag{15}$$

with

- modal stiffness k_j [N/m]
- modal mass m_j [kg]
- modal frequency $w_{e,j} = \sqrt{k_j / m_j}$ [rad/s]
- modal controllability factor l_j [–]

When one would think of building up the collocated response, starting with the static crosstalk, and subsequently adding consecutive modes, each mode *j* would have three effects on the response (see the upper plots in figure 4):

- a resonance appears at $w_{e,j}$
- the apparent crosstalk beyond $w_{e,j}$ increases to $k_{beyond\ w_{e,j}} = k_0 + \sum_{i=1..j} l_i^2 k_i$ (16)
- anti-resonance frequencies appear at (or shift to) the frequencies for which the newly added term crosses the response built up so far.

Due to the collocation, the modes add up in such a way that all anti-resonance frequencies are located in-between the resonance frequencies, constituting the alternating pole-zero-pattern. In section 3 it was already indicated that damping *robustness* is due to the *alternation* of the poles and zeros. Damping *performance* however can be shown to depend on the *separation* of the poles and zeros: vibration modes corresponding to poles that do not have zeros nearby, are in general well controllable, and can well be damped. Separation of the zero corresponding to the pole for mode *j* can be shown to determined to a large extent by the factor $l_j^2 k_j$ in eq. (15).

A close look at figure 4, in combination with the structures in figure 3, reveals that the actuator location in relation to the mode shapes determines to a large extent the location of the zeros. As the zeros constitute the end-points of the root-locus, the actuator location to a large extent determines the controllability of the vibration modes. In this respect a Smart Disc *beneath* the masses can be seen to be well suited to control the lowest vibration mode, corresponding to the *in-phase* movement of the masses. A Smart Disc *in-between* both masses, on the other hand, is well suited to control the higher vibration mode, corresponding to the *out-of-phase* movement of the masses.

Controllability however does not solely depend on the actuator location. It also depends to a large extent on the amount of *crosstalk*. This insight is illustrated in figure 5, which shows the collocated responses and root-loci for two different values of the static contribution k_0. High crosstalk can be seen to lead to zeros which *all* are very close to the poles. In general it can be said that, irrespective of the actuator-sensor-pair location,

- high crosstalk implies bad controllability and low achievable damping, and
- low crosstalk leads to better controllability and higher achievable damping.

In the next section it will be shown that crosstalk, and in turn its influence on the controllability, will appear to be of crucial importance when using a single piezoelectric element, i.e. a 'self-sensing actuator', for either passive or active vibration damping.

5 Self-sensing actuator

In section 2 several equations were given to describe the behaviour of a piezoelectric actuator or sensor. For the sensor the assumption was made that the charge could be kept at zero. In case a *single* piezoelectric element is used both as sensor and actuator, this prerequisite obviously is violated, as in order to generate a displacement, an electrical charge needs to be supplied. The measured voltage however is directly related to this charge, through the capacitance of the piezoelectric element. In comparison to a collocated but separate actuator-sensor-pair, the collocated response for a *self-sensing actuator* (SSA) thus is affected by an additional term (Anderson and Hagood 1994):

$$H_{SSA}(s) = \frac{U_{SSA}}{q_{SSA}}(s) = g_{SSA} H_{mech}(s) g_{SSA} + \left(C_{el}^{F}\right)^{-1} \tag{17}$$

Due to the fact that this additional capacitance-related term is relatively large, the zeros of the collocated response are very close to the poles, and controllability for a self-sensing actuator is very bad.

The zeros of the response given by eq. (17) can be shown to correspond to the resonance frequencies of the mechanical structure for shorted electrodes, in which case the stiffness of the actuator is given by k_m^U. The poles on the other hand are known to correspond to the resonance frequencies of the mechanical structure for open electrodes, in which case the stiffness of the actuator is given by eq. (3). This implies that the controllability of a

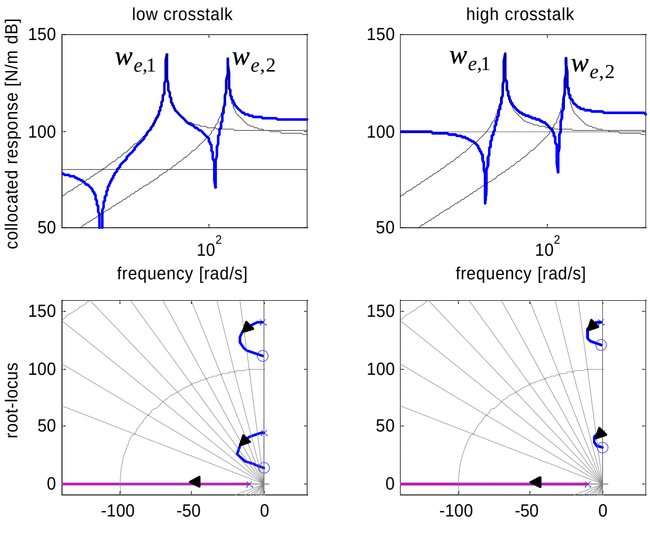

Figure 5 Illustration of the influence of crosstalk on the collocated frequency response and the IFF-root-locus

self-sensing actuator largely depends on the role the actuator stiffness plays in the entire mechanical structure. In this respect a self-sensing actuator may be considered equivalent to a highly preloaded collocated actuator-sensor-pair, as in both cases IFF-damping is very poor.

Nevertheless it is interesting to note that, for a self-sensing actuator, IFF may be implemented *passively*, as the control law relates an electrical charge to the integral of a voltage of the same piezoelectric element:

$$q_{SSA} = \frac{K}{s} U_{SSA} \tag{18}$$

Consequently the control law imposes a linear relationship between the current through the piezoelectric element and the voltage across it:

$$i_{SSA} = K U_{SSA} \tag{19}$$

In short: the control law may be implemented by means of a resistive shunt valued $R = 1/K$. As a resistive shunt inherently dissipates energy, passive control as such is very robust. For obvious reasons, i.e., the lack of controllability, passive damping performance is very poor: achievable relative damping levels in general do not exceed $z = 0.005$ (Hagood and Von Flotow 1991).

In order to improve passive damping performance, the piezoelectric element could also be shunted by means of a passive electric circuit with an impedance that is carefully tuned to the frequency of the vibrations to be damped. The passivity of such an implementation still guarantees robust stability, while the relative damping may in practice be increased to $z = 0.02$. Performance however is far from robust as sub-optimal tuning largely degrades the effectiveness of the passive control scheme (Preumont 1997). The next section briefly deals with an intuitive method to improve performance in the case of *active* control.

6 Crosstalk compensation

When it is known that a large amount of crosstalk is the main cause of poor controllability, the achievable damping may be improved by compensating for the crosstalk before applying IFF. Crosstalk compensation, which can only be performed *actively*, as such is an intuitive method to shift the zeros (of the loop to be closed) away from the poles. The robustness of the control scheme is not endangered that much, as the favourable alternating pole-zero-pattern is preserved as long as it is assured that the net crosstalk remains positive. In theory, care thus only needs to be taken to prevent 'over-compensation', which would result in a net negative crosstalk and in the possibility of closed-loop instability (Anderson and Hagood 1994; Carabelli and Tonoli 2000).

In practice, when using piezoelectric elements, things get more complicated, due to the fact that the capacitance of a piezoelectric element suffers from hysteresis and other non-linearities. The relations between U and q, and between U and x in eq. (1), are therefore non-linear, and so is the crosstalk. As a consequence, improvement of the achievable damping by means of *linear* crosstalk compensation is limited in practice (Waanders 1991; Anderson and Hagood 1994; Carabelli and Tonoli 2000).

Experiments performed so far indeed have shown the beneficial effects of crosstalk compensation as well as the hysteretic nature of the crosstalk. Crosstalk compensation in these experiments turned out to be necessary because the restriction of the analysis of the actuator-sensor-pair to a single direction (refer section 2) was not allowed. Nevertheless a relative damping level as high as $z = 0.20$ was achieved. The main lesson learned from the experiments is that an actuator-sensor-pair should be designed properly, so as to assure 'correct' collocation and low crosstalk (Holterman and De Vries 2001).

7 Conclusions

1. A properly designed collocated actuator-sensor-pair enables excellent and robust active damping (section 3), as long as sufficient controllability is assured (section 4).
2. A self-sensing actuator (i.e., an actively controlled single piezoelectric element) also enables excellent active damping, though less robust, due to the prerequisite of correct crosstalk compensation (section 5, 6).
3. With respect to controllability issues, a badly designed collocated actuator-sensor-pair may be considered equivalent to a self-sensing actuator, as in both cases cross-talk compensation is needed to achieve a reasonable damping level (section 5, 6).
4. A self-sensing actuator without crosstalk compensation may as well be used passively, but only moderate damping values can be obtained (section 5).

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