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Modeling and verifying non-linearities in heterodyne displacement interferometry

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Abstract

The non-linearities in a heterodyne laser interferometer system occurring from the phase measurement system of the interferometer and from non-ideal polarization effects of the optics are modeled into one analytical expression which includes the initial polarization state of the laser source, the rotational alignment of the beam splitter along with different transmission coefficients for polarization states and the rotational misalignment of the receiving polarizer. The model is verified using a Babinet Soleil Compensator allowing a common path for both polarization states and thereby reducing the influence of the refractive index of air. The verification shows an agreement of the model with measurements with a standard deviation of 0.2 nm. With the use of the model it is confirmed that the mean of two polarizer receivers can reduce the effect of non-linearity. However, depending on the accuracy of the polarizer angles, a second-order non-linearity remains. Also the effect of rotational misalignment of the beam splitter can not be reduced in this way.

Keywords: Interferometry; Non-linearities

1. Introduction

The field of interferometry tends towards more accurate measurements with ever increasing resolution. The principal limitations of laser interferometer systems are in the photonic noise and the residual non-linearities which are inherent in periodic signals in laser interferometers. These non-linearities repeat each fraction of a wavelength. They are present in the phase measurement system of the interferometer itself [1], but they can also result from the optics used: beam splitters, retardation plates used with plane mirrors and corner cubes can influence the polarization state in interferometers using polarizing optics, or influence the contrast in interferometers with non-polarizing optics. In this paper, we deal with the influence of optics in a heterodyne laser interferometer, using polarizing optics, in the configuration shown in Fig. 1.

These laser configurations are widely used for geometrical measurements. In this field the required resolution and accuracy improve continuously. The modeling of these non-linearities therefore is important to understand the influence of different sources on the non-linearity and in any future compensation for these errors. Until now research done on the influence of sources on non-linearity [2,3] was based on the calculation of different influences separately or a combination of influences from the beam splitter [4]. Other research aimed on the direct measurement and elimination of first- and second-order harmonics in the measurement signal as a result of different separate influences [5,6]. We give a formula including the polarization state of the laser, the rotational misalignment of the beam splitter along with different transmission coefficients and the rotational misalignment of the mixing polarizer. We assume all other properties of the interferometer ideal, which implies ideal corner cubes and an ideal coupling of these parts concerning the polarization states.

2. Theory of heterodyne laser interferometry

The laser shown in Fig. 1 consists of a laser source with two orthogonal polarized beams with a different frequency (f_1 and f_2). These two beams can be represented as:

$$E_1 = E_0 \sin(2\pi f_1 t + \phi_{01}) \quad (1)$$

$$E_2 = E_0 \sin(2\pi f_2 t + \phi_{02}) \quad (2)$$

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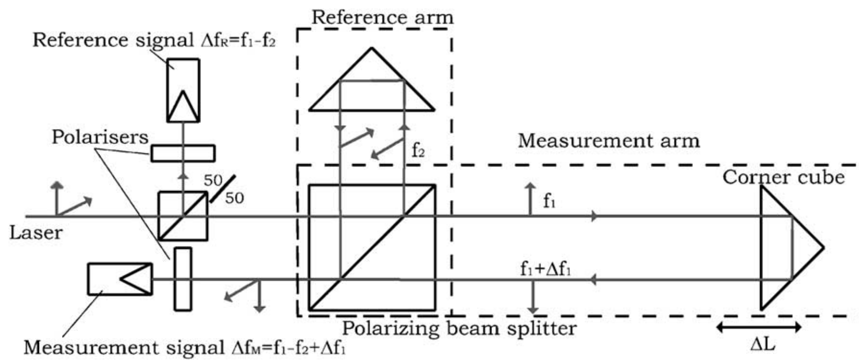

Fig. 1. Schematical representation of the principle of a heterodyne laser interferometer.

A non-polarizing beam splitter divides the beam in two parts: one reference part and one measurement part. The reference part passes a combining polarizer and a detector resulting in an interference signal of which only the alternating current is measured due to the use of a band-pass filter, eliminating both the dc component and optical frequencies. This signal can be represented by:

$$I_{\text{ref}} = \tfrac{1}{2}E_0^2[\cos(2\pi(f_1 - f_2)t + (\phi_{01} - \phi_{02}))] \quad (3)$$

The measurement part is divided by a polarizing beam splitter in a reference arm and in a measurement arm. The signal in the measurement arm receives a Doppler shift as a result of the moving corner cube, resulting in a phase shift of the measurement signal compared to the reference signal [7]:

$$\Delta\phi_1 = 2\pi\,\Delta f_1 t = \frac{4\pi n\,\Delta l}{\lambda_1} \quad (4)$$

where λ_1 represents the wavelength of the light reflected by the mirror and n the refractive index of air in the measurement path. With a polarizer the signal from the reference arm and the signal from the measurement arm are combined and result in an interference pattern. From this interference pattern the measurement signal is constructed containing the phase information and therefore containing the displacement of the corner cube. In an ideal laser interferometer E_1 enters the measurement arm and E_2 enters the reference arm and the detected signal can be represented by:

$$I_{\text{meas}} = \tfrac{1}{2}E_0^2[\cos(2\pi(f_1 - f_2)t + (\phi_{01} - \phi_{02}) + \Delta\phi_1)] \quad (5)$$

where f_1 and f_2 are the frequency of measurement and reference arm, ϕ_{01} and ϕ_{02} the initial phase of both frequencies and $\Delta\phi_1$ is the phase difference as a result of the target movement. The measurement with a laser interferometer is based on measuring the phase difference between the measurement signal (5) and the reference signal (3). In a common laser interferometer the separation between both arms and therefore both polarization states is not ideal, part of the measurement frequency enters the reference arm and part of the reference frequency enters the measurement arm and gets a Doppler shift also. This results in an extra phase term in the measurement signal which now can be represented as:

$$I_{\text{measnl}} = \tfrac{1}{2}E_0^2[\cos(2\pi(f_1 - f_2)t + (\phi_{01} - \phi_{02}) + \Delta\phi_1 + \Delta\phi_{\text{nonlin}})] \quad (6)$$

The calculation in the interferometer however uses only one frequency resulting in non-linearities in the measurement results. The extra phase term therefore represents the non-linearity of the system.

3. Calculating the non-linearities

As earlier studies have proven, a number of different influences on non-linearities in laser interferometers exist [2–4,8–10]. The importance of the influence of elliptical and non-orthogonal polarized light on the accuracy of a heterodyne laser interferometer was described by [11]. We will now formulate an analytical formula which includes all previous studied influences on the non-linearity:

- Elliptical polarized laser beams.
- Non-orthogonal polarized laser beams.
- Rotational error in the alignment of laser and beam splitter.
- Rotational error in the alignment of the mixing polarizer.
- Different transmission coefficients in the beam splitter.

The phase measuring electronics are not considered in this paper. The errors mentioned above can be represented schematically as in Fig. 2. The ellipticity of the laser polarization beams is defined as $d\varepsilon_1$ and $d\varepsilon_2$, the non-orthogonality and rotation of the laser referred to the beam splitter are both represented by α and β, i.e., for a rotational error of orthogonal polarized light $\beta = -\alpha$. The transmission coefficients are represented as χ and ξ. Finally the rotation angle of the polarizer compared to $\pi/4$ is referred to as θ.

Elliptical polarized light emitted by the laser can be represented as:

$$E_1 = E_0[\cos(\tfrac{1}{2}d\varepsilon_1)\sin(2\pi f_1 t + \phi_{01}) - \sin(\tfrac{1}{2}d\varepsilon_2)\cos(2\pi f_2 t + \phi_{02})] \quad (7)$$

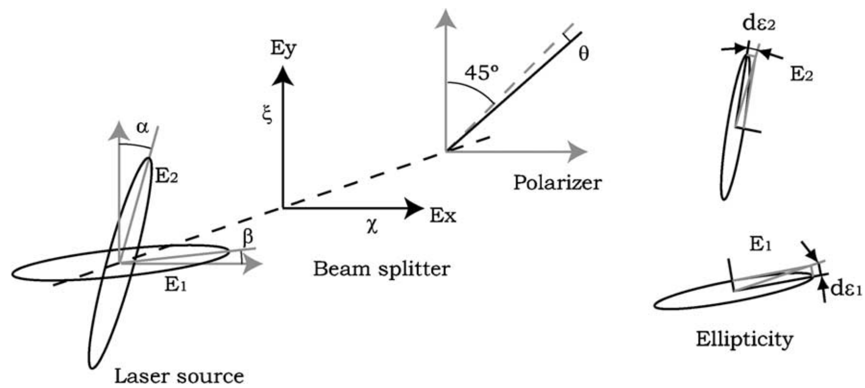

Fig. 2. Schematical representation of the parameters influencing the non-linearity in a laser interferometer.

$$E_2 = E_0[\sin(\tfrac{1}{2}d\varepsilon_1)\cos(2\pi f_1 t + \phi_{01}) - \cos(\tfrac{1}{2}d\varepsilon_2)\sin(2\pi f_2 t + \phi_{02})] \quad (8)$$

where $d\varepsilon_1$ and $d\varepsilon_2$ represent the ellipticity as shown in Fig. 2.

For the simplicity of further formulas this can be represented as:

$$E_1(x_1, x_2) = E_0[\cos(\tfrac{1}{2}d\varepsilon_1)\sin(x_1) - \sin(\tfrac{1}{2}d\varepsilon_2)\cos(x_2)] \quad (9)$$

$$E_2(x_1, x_2) = E_0[\sin(\tfrac{1}{2}d\varepsilon_1)\cos(x_1) - \cos(\tfrac{1}{2}d\varepsilon_2)\sin(x_2)] \quad (10)$$

After transmission through the beam splitter the two beams can be represented as:

$$E_x = \xi[\cos\beta E_1(x_1+\phi_1, x_2+\phi_1) + \sin\alpha E_2(x_1+\phi_1, x_2+\phi_1)] \quad (11)$$

$$E_y = \chi[\sin\beta E_1(x_1+\phi_2, x_2+\phi_2) + \cos\alpha E_2(x_1+\phi_2, x_2+\phi_2)] \quad (12)$$

where ξ and χ are the transmission coefficients in, respectively, x and y direction of the beam splitter and α and β represent the non-orthogonality and rotational alignment error. The phase retardance of the two beams transmitted by the polarizing beam splitter are indicated as ϕ_1 and ϕ_2, respectively.

The intensity transmitted through a rotated polarizer can be represented as:

$$I = \left[E_x\cos\left(\frac{\pi}{4}-\theta\right) + E_y\sin\left(\frac{\pi}{4}-\theta\right)\right] \quad (13)$$

where θ represents the angle compared to $\pi/4$.

Substitution of these formulas into each other and simplifying it into one cosine term gives the formula for the intensity on the detector in the form of formula (6) but with a different amplitude. However, as mentioned before the measurement of the interferometer is based on a phase measurement. The phase of the resulting formula therefore includes the non-linearity.

4. The full formula

The non-linearities resulting from a non-orthogonal, elliptical polarized laser light source passing a rotational misaligned beam splitter with different transmission coefficients and a rotated polarizer can be calculated with the use of formula (14):

$$\Delta\phi_{\text{nonlin}} = -\arctan\frac{A + B\sin(2\theta) + C\cos(2\theta)}{D + E\sin(2\theta) + F\cos(2\theta)} \quad (14)$$

where

$$\begin{aligned}
A = {} & [-[\xi^2\sin^2\beta + \chi^2\cos^2\beta]\cos(\tfrac{1}{2}d\varepsilon_1)\sin(\tfrac{1}{2}d\varepsilon_2) \\
& - [\xi^2\cos^2\alpha + \chi^2\sin^2\alpha]\sin(\tfrac{1}{2}d\varepsilon_1)\cos(\tfrac{1}{2}d\varepsilon_2)] \\
& \times \cos\Delta\phi + [\xi^2\cos\alpha\sin\beta + \chi^2\sin\alpha\cos\beta] \\
& \times \cos(\tfrac{1}{2}d\varepsilon_1 + \tfrac{1}{2}d\varepsilon_2)\sin\Delta\phi, \\
B = {} & [[\xi^2\sin^2\beta - \chi^2\cos^2\beta]\cos(\tfrac{1}{2}d\varepsilon_1)\sin(\tfrac{1}{2}d\varepsilon_2) \\
& + [\xi^2\cos^2\alpha - \chi^2\sin^2\alpha]\sin(\tfrac{1}{2}d\varepsilon_1)\cos(\tfrac{1}{2}d\varepsilon_2)] \\
& \times \cos\Delta\phi + [-\xi^2\cos\alpha\sin\beta + \chi^2\sin\alpha\cos\beta] \\
& \times \cos(\tfrac{1}{2}d\varepsilon_1 + \tfrac{1}{2}d\varepsilon_2)\sin\Delta\phi, \\
C = {} & \xi\chi[\cos\beta\sin\beta\cos(\tfrac{1}{2}d\varepsilon_1)\sin(\tfrac{1}{2}d\varepsilon_2) \\
& \times [1 - \cos 2\Delta\phi] + \sin\alpha\sin\beta\cos(\tfrac{1}{2}d\varepsilon_1) \\
& \times \cos(\tfrac{1}{2}d\varepsilon_2)\sin 2\Delta\phi - \cos\alpha\cos\beta\sin(\tfrac{1}{2}d\varepsilon_1) \\
& \times \sin(\tfrac{1}{2}d\varepsilon_2)\sin 2\Delta\phi - \sin\alpha\cos\alpha\sin(\tfrac{1}{2}d\varepsilon_1) \\
& \times \cos(\tfrac{1}{2}d\varepsilon_2)[1 + \cos 2\Delta\phi]], \\
D = {} & [[\xi^2\sin^2\beta + \chi^2\cos^2\beta]\cos(\tfrac{1}{2}d\varepsilon_1)\sin(\tfrac{1}{2}d\varepsilon_2) \\
& + [\xi^2\cos^2\alpha + \chi^2\sin^2\alpha]\sin(\tfrac{1}{2}d\varepsilon_1)\cos(\tfrac{1}{2}d\varepsilon_2)] \\
& \times \sin\Delta\phi + [\xi^2\cos\alpha\sin\beta + \chi^2\sin\alpha\cos\beta] \\
& \times \cos(\tfrac{1}{2}d\varepsilon_1 + \tfrac{1}{2}d\varepsilon_2)\cos\Delta\phi,
\end{aligned}$$

$$
\begin{aligned}
E = {} & [[-\xi^2 \sin^2\beta + \chi^2 \cos^2\beta] \cos(\tfrac{1}{2}d\varepsilon_1) \sin(\tfrac{1}{2}d\varepsilon_2) \\
& + [-\xi^2 \cos^2\alpha + \chi^2 \sin^2\alpha] \sin(\tfrac{1}{2}d\varepsilon_1) \\
& \times \cos(\tfrac{1}{2}d\varepsilon_2)] \sin\Delta\phi + [-\xi^2 \cos\alpha \sin\beta \\
& + \chi^2 \sin\alpha \cos\beta] \cos(\tfrac{1}{2}d\varepsilon_1 + \tfrac{1}{2}d\varepsilon_2) \cos\Delta\phi, \\
F = {} & \xi\chi[\cos\beta \sin\beta \cos(\tfrac{1}{2}d\varepsilon_1) \sin(\tfrac{1}{2}d\varepsilon_2) \sin 2\Delta\phi \\
& + \cos\alpha \cos\beta[\cos(\tfrac{1}{2}d\varepsilon_1) \cos(\tfrac{1}{2}d\varepsilon_2) \\
& - \sin(\tfrac{1}{2}d\varepsilon_1) \sin(\tfrac{1}{2}d\varepsilon_2) \cos 2\Delta\phi] + \sin\alpha \sin\beta \\
& \times [-\sin(\tfrac{1}{2}d\varepsilon_1) \sin(\tfrac{1}{2}d\varepsilon_2) + \cos(\tfrac{1}{2}d\varepsilon_1) \\
& \times \cos(\tfrac{1}{2}d\varepsilon_2) \cos 2\Delta\phi] + \sin\alpha \cos\alpha \sin(\tfrac{1}{2}d\varepsilon_1) \\
& \times \cos(\tfrac{1}{2}d\varepsilon_2) \sin 2\Delta\phi]
\end{aligned}
$$

Simplifying this formula with $\alpha = \beta = \theta = d\varepsilon_2 = 0$ and $\xi = \chi = 1$, which means an ideal interferometer except for one elliptical polarized laser beam, gives the following equation:

$$\Delta\phi_{\text{nonlin}} = -\arctan\left(\frac{-\sin((\frac{1}{2})d\varepsilon_1)\cos(\Delta\phi)}{\sin((\frac{1}{2})d\varepsilon_1)\sin(\Delta\phi) + \cos((\frac{1}{2})d\varepsilon_1)}\right) \quad (15)$$

This is equal to Eq. (13) of Hou and Wilkening [2] where they use $\rho = -(1/2)d\varepsilon_1$. Another simplification follows from substituting $\beta = -\alpha$ and $d\varepsilon_1 = d\varepsilon_2 = \theta = 0$ and $\xi = \chi = 1$ which means a rotational misalignment of the beam splitter gives:

$$\Delta\phi_{\text{nonlin}} = -\arctan\left(\frac{-\sin^2\alpha \sin(2\Delta\phi)}{\cos^2\alpha - \sin^2\alpha \cos(2\Delta\phi)}\right) \quad (16)$$

Rewriting of this formula results in Eq. (11) found by De Freitas and Player [3]. So the mentioned different cases modeled by [2,3] follow exactly from a simplification of the full formula (14).

5. Modeling of non-linearities

With the use of formula (14) combinations of different sources of non-linearities were modeled. In this section we show you some of the results. In Fig. 3 the non-linearities are shown resulting from different angles of rotational misalignment of the beam splitter (a) and from different angles of the polarizer in front of the measurement detector, combined with a constant rotational angle of the beam splitter (b). All other parameters are assumed ideal ($\beta = -\alpha$, $\xi = \chi = 1$, $d\varepsilon_1 = d\varepsilon_2 = 0$). From this figure it can be concluded that the rotation of the receiving polarizer determines the period of the non-linearity as mentioned by [4]. A rotation of the polarizer equal to the angle of the rotated beam splitter results in a period of $\lambda/4$ and a polarizer aligned ideal to the orientation of the laser results in a period of $\lambda/2$. Both periods were observed earlier by [12]. This solves the discrepancy between authors which mention a period of $\lambda/4$ [3] and those who observed a period of $\lambda/2$ [8]. Other modeling showed that the effect was a result of equaling the projected amplitudes of the measurement arm and the reference arm on the measurement polarizer. If in fact the transmission amplitudes are $\xi = 0.5$ for one arm and $\chi = 1$ for the other arm, the measurement polarizer should have an angle of 18.4° with the 45° axis to reduce the effect of the first-order non-linearity, which equals 45° − arctan(0.5).

Since the principal limitations of the laser interferometer are in the polarization state of the laser source itself, the non-orthogonality and ellipticity are also modeled. In Fig. 4 the effect of non-orthogonality of the laser beams on the non-linearity can be seen. The result of rotating the polarizer now towards the orthogonality direction is a reduction of the amplitude of the non-linearity. The period remains $\lambda/2$.

A second influence coming directly from the laser source is the ellipticity of the laser polarization. An interesting effect of elliptical polarized light is elliptical polarized light emerging from the laser in combination with a rotational alignment error. The results are shown in Fig. 5. We suppose the polarizer in 45° with the beam splitter axes. As can be seen as a result of the elliptical polarized laser light the mean of the non-linearity is no longer zero.

Certain laser interferometers contain a polarizer and receiver included in the laser head. As a result the polarizer will no longer be in 45° with the beam splitter axes, but the polarizer will still be aligned 45° with the laser axes. The non-linearity resulting from a combination of elliptical

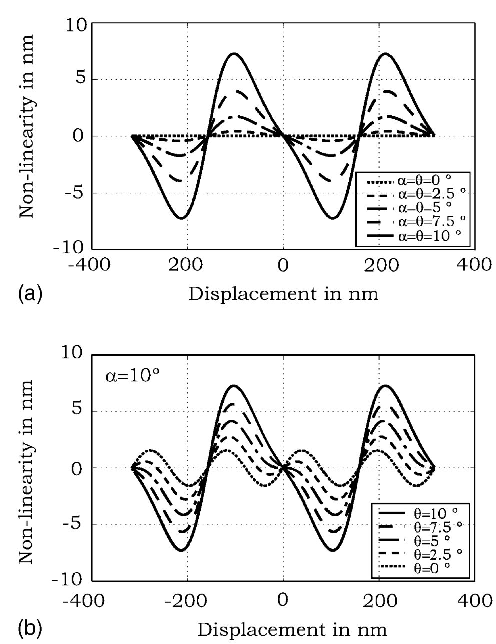

Fig. 3. Non-linearity resulting from different rotation angles of the beam splitter (a) and from different rotation angles of the polarizer with a constant rotation angle ($\alpha = 10°$) of the beam splitter (b).

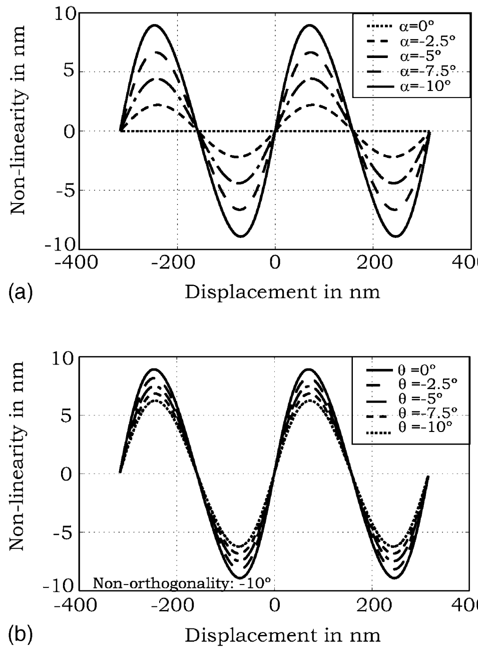

Fig. 4. Non-linearity resulting from different angles of non-orthogonality (a) and from different rotation angles of the polarizer with a constant non-orthogonality ($\alpha = -10°$, $\beta = 0°$) (b).

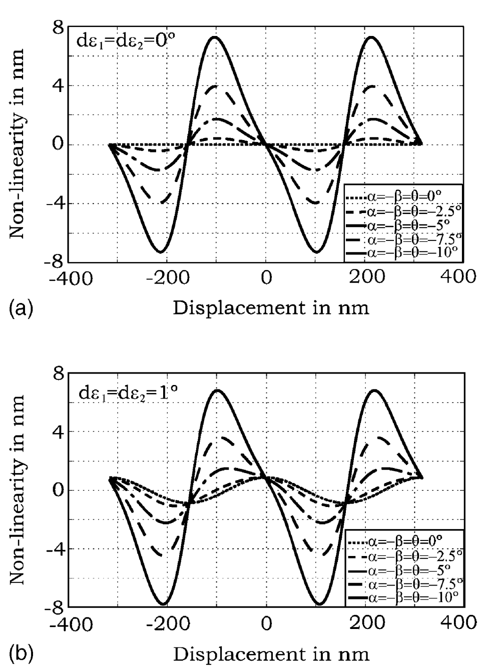

Fig. 5. Non-linearity resulting from different angles of rotation of the beam splitter with linear orthogonal polarized laser light (a) and from different rotation angles of the beam splitter with an elliptical polarized light ($d\varepsilon_1 = d\varepsilon_2 = 1°$) (b). The alignment of the polarizer is in 45° with the beam splitter axes.

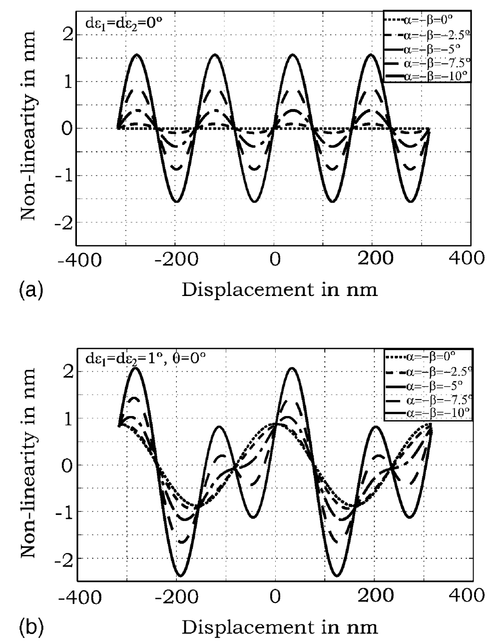

Fig. 6. Non-linearity resulting from different angles of rotation of the beam splitter with linear orthogonal polarized laser light (a) and from different rotation angles of the beam splitter with an elliptical polarized light ($d\varepsilon_1 = d\varepsilon_2 = 1°$) (b), with the alignment of the polarizer 45° compared to the laser polarization.

polarized light and a rotated beam splitter can be seen in Fig. 6. In Fig. 6 it can be seen that the elliptical polarization introduces an increasing first-harmonic non-linearity (period $\lambda/2$). To show this effect more clearly the Fourier transformation of the signals in Fig. 6 is shown in Fig. 7.

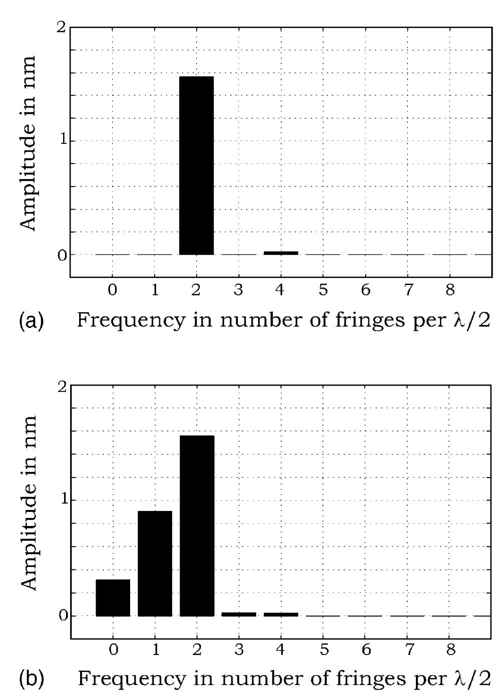

Fig. 7. The amplitude spectrum of the maximum signals in Fig. 6.

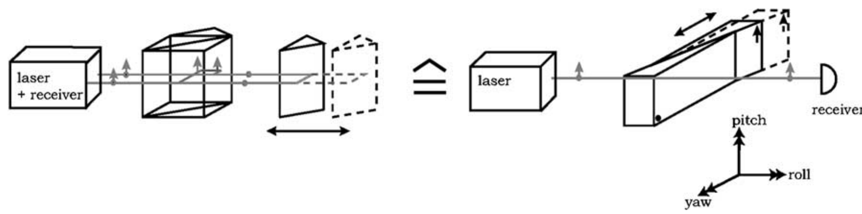

Fig. 8. Schematical representation of the simulation of the interferometer optics with a Babinet Soleil Compensator.

6. Verification of the model using a Babinet Soleil Compensator

The model given in Section 4 combined different theories resulting from previous publications of various authors. To verify the theory in practice we used a Babinet Soleil Compensator, replacing the beam splitter-corner cube optics of the modeled interferometer (see Fig. 8).

The Babinet Soleil Compensator provides a common path for both polarization components, and therefore cancels out any influence of the refractive index of air. It also provides a convenient way to apply an optical path difference of parts of a wavelength. This optical path difference is produced due to a difference in refractive index for two orthogonal polarization states. The phase difference introduced between both polarization axes can be represented as [13]:

$$\Delta\phi = 2\pi \left| \frac{n_\perp}{\lambda_{\perp 0}} - \frac{n_\parallel}{\lambda_{\parallel 0}} \right| d \tag{17}$$

where $\lambda_{\perp 0}$ and $\lambda_{\parallel 0}$ are the vacuum wavelengths of both polarization states, $n_\perp$ and $n_\parallel$ are the refractive indices for both polarization arms and d represents the thickness of the Babinet Soleil Compensator. By varying the thickness of the compensator (through movement of the wedges compared to each other, see Fig. 8) the optical path difference between both polarization states can be varied.

The test setup consists of a heterodyne laser source (HP type 5519) placed firmly on an optical bench, a Babinet Soleil Compensator which can be rotated around its roll-axis and a receiver (HP type 10780B) which can be rotated around its roll-axis also. The receiver was read out with a PC-card (HP type 10887A) with help of own software which reads the card at maximum resolution and integrates over 10 measurements.

An important aspect for the verification of the model with use of a Babinet Soleil Compensator is the repeatability of the measurements. This repeatability was tested in three sessions done at different times during 1 day. The results are shown in Fig. 9. It shows a standard deviation of 0.1 nm which is sufficient for the tests to be done.

To ensure a correct measurement of non-linearities the Babinet Soleil scale was calibrated, using a second Babinet Soleil Compensator. The procedure consisted of carrying out a measurement as shown in Figs. 10 and 11 at different initial phase shift (as generated by the second Babinet Soleil Compensator) and averaging the results. Initially for both Babinet

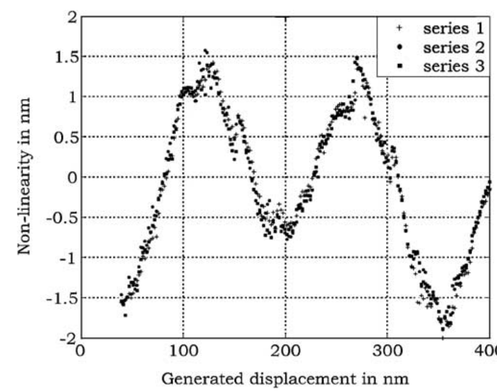

Fig. 9. Result of the repeatability test for the Babinet Soleil Compensator.

Soleil Compensators the optical displacement was calibrated against the mechanical displacement at 0, $\lambda/2$ and λ. This was done using two polarizers, one between laser and Babinet Soleil Compensator, and one between compensator and a dc detector, at different orientations of 0° and 90°. Then four measurements of the non-linearity of an optimal aligned laser–Babinet Soleil receiver configuration, like in Fig. 8, were done with the second Babinet Soleil Compensator at a phase shift representing $0, \lambda/4, \lambda/2$ and $3\lambda/4$. At these four initial phase shifts the Babinet Soleil Compensator used for further measurements was displaced within the same range as during the measurements. Averaging these four

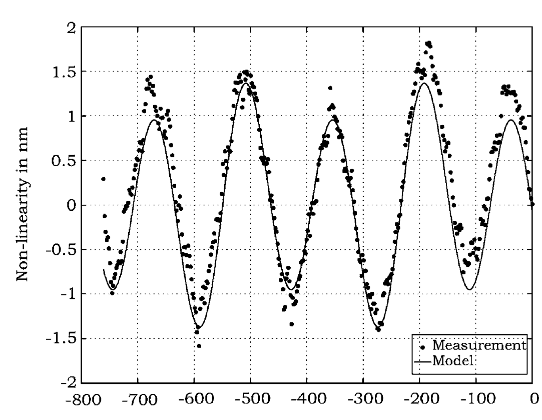

Fig. 10. Theoretical and experimental results of the measurement of the non-linearity in an interferometer with a rotated beam splitter ($\alpha = -8.4°$, $\beta = 8.5°$) and a rotated polarizer ($\theta = -9.4°$). Further variables used in the model were: $\xi = 0.72$, $\chi = 0.71$, $d\varepsilon_1 = d\varepsilon_2 = 0.004°$.

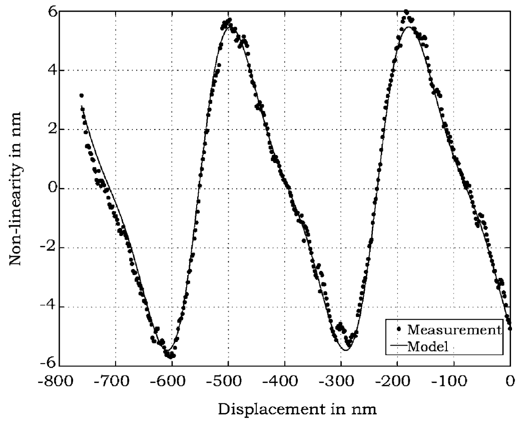

Fig. 11. Theoretical and experimental results of the measurement of the non-linearity in an interferometer with a rotated beam splitter ($\alpha = -8.4^\circ$, $\beta = 8.5^\circ$) and a rotated polarizer ($\theta = -1.0^\circ$). Further variables used in the model were: $\xi = 0.72$, $\chi = 0.71$, $d\varepsilon_1 = d\varepsilon_2 = 0.004^\circ$.

measurements results in the calibration as shown in Fig. 12 and was used to correct the measurements of non-linearities.

In the measurement setup there are two controlled variables: the rotation angle of the Babinet Soleil Compensator (representing α and $-\beta$) and the rotation angle of the receiving polarizer (representing θ). All other parameters present in formula (14) are kept constant. To be able to model the interferometer correctly the non-orthogonality and ellipticity of the laser polarizations and the transmission coefficients of the Babinet Soleil Compensator were measured with use of a polarizer and a spectrum analyzer. The ellipticity for the laser polarizations appeared to be 0.004° for E_1 and E_2. The non-orthogonality was 1.0°. The transmission coefficients of the Babinet Soleil were 0.72 for horizontal component and 0.71 for the vertical component. These values were used for further simulations comparing the model with the measurements with the Babinet Soleil Compensator.

In Figs. 10 and 11 two results are presented of the measurement with use of a Babinet Soleil Compensator together with the result of the analytical model.

Both figures show an excellent correspondence between the theory and the measurements resulting in a standard deviation of the model compared to the measurements of only 0.2 nm.

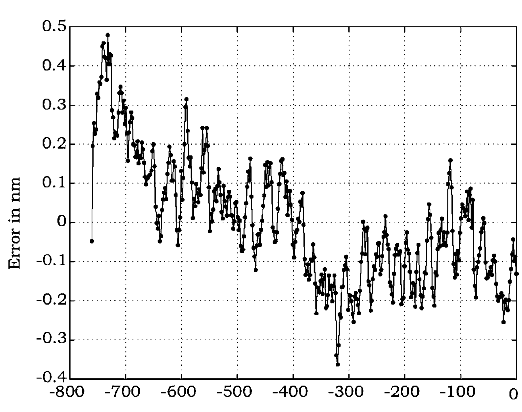

Fig. 12. Calibration result for the Babinet Soleil Compensator.

7. Eliminating the non-linearities

With the use of the model the elimination of the non-linearities in the signal with use of different measurement detectors with different polarization angles and averaging both signals, as mentioned by [2] and used by [14] can be modeled. With these simulations it can be shown that the first-order non-linearities occurring due to non-ideal polarization state of the laser are canceled out, however the arithmetical mean of the two interference phases does not become zero as can be seen in Fig. 13a, a second-order harmonic remains. In Fig. 14b one can see that the mean of a periodic non-linearity with variable amplitude becomes a periodic non-linearity with constant amplitude. However, when the polarizers are not aligned in 45° with the beam splitter axes, or when the angle between the two polarizers is not exactly 90° the resulting mean becomes a periodic non-linearity with variable amplitude. This effect can be seen in Fig. 13b. In Fig. 14 the effect of averaging over

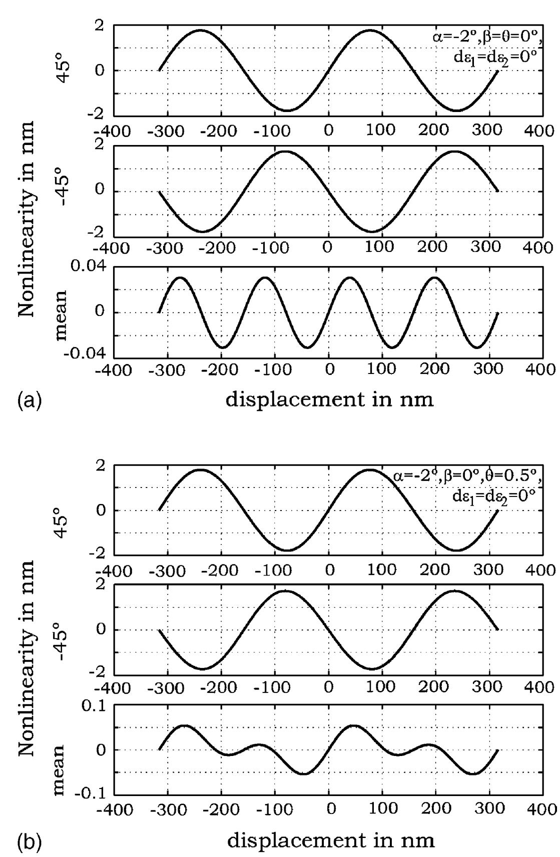

Fig. 13. The effect of a non-orthogonal laser and ideal aligned polarizer combination (a) and the effect of a non-orthogonal laser in combination with a misaligned polarizer combination of 0.5° (b). Part (b) equals the effect of an angle of 91° between the two polarizers and the first at 45° with the beam splitter axes.

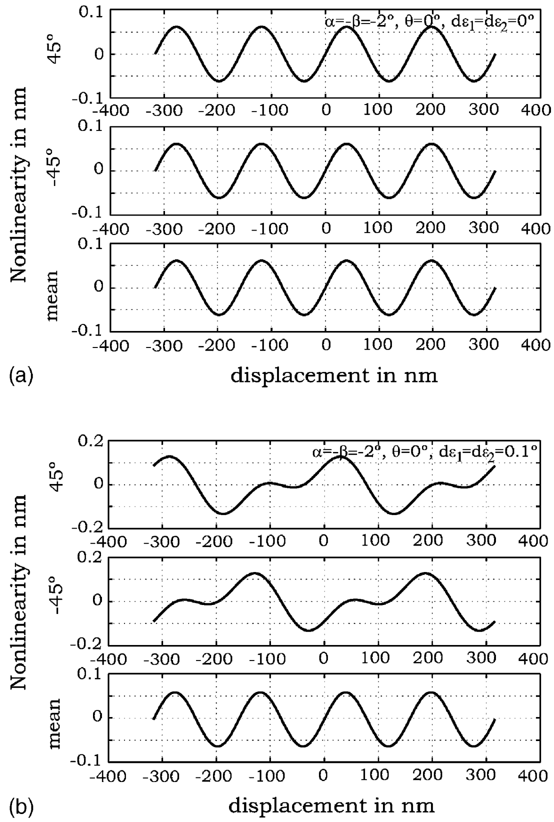

Fig. 14. The effect of an increasing ellipticity of the laser source superposed on the rotation of the beam splitter by 2° on the mean of the non-linearities on two detectors.

two detectors is shown for increasing ellipticity of the laser beam in combination with a constant rotation angle of the beam splitter. It can be seen that the non-linearity resulting from a rotated beam splitter is not canceled out. The effect of ellipticity of the laser beam is canceled out. The ellipticity results in an offset in the mean signal, however this does not result in measurement errors since this offset is a constant factor.

Concluding: non-linearities resulting from rotations of the beam splitter and a polarizer remaining under 45° with the beam splitter axes, which introduce a pure second-order non-linearity, can not be canceled out with this technique, as can also be seen in Fig. 14. First-order non-linearities resulting from a displaced polarizer are not canceled out either, therefore care must be taken that the polarizers are in 45° with the beam splitter axes.

8. Conclusions

- We succeeded in the analytical modeling of a basic heterodyne laser interferometer including the influence of the laser source and the alignment of the optics with an agreement of 0.2 nm.
- With the use of this formula non-linearities can be predicted by measuring the polarization states of the beam and optics instead of difficult integral calibrations.
- The effect of non-linearities produced by non-orthogonal laser sources or elliptical sources can be reduced with the use of two receiving polarizers under 90° and averaging on both signals.
- The non-linearity resulting from a rotated beam splitter can not be canceled out.

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